Pancyclic arcs in Hamiltonian cycles of tournaments

Michel Surmacs

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Introduction

Theorem (Rédei, 1934)

Every tournament contains a Hamiltonian path.

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Theorem (Camion, 1959)

Every strong tournament contains a Hamiltonian cycle.

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Theorem (Rédei, 1934)
Every tournament contains a Hamiltonian path.

Theorem (Camion, 1959)
Every strong tournament contains a Hamiltonian cycle.

Theorem (Thomassen, 1980)
For every $k \geq 1$ there are infinitely many tournaments with precisely k Hamiltonian cycles.

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Theorem (Thomassen, 1980)

Every 4-strong tournament is strongly Hamiltonian-connected.

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Every arc of a 3-strong tournament is contained in a Hamiltonian cycle.

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Every 4-strong tournament is strongly Hamiltonian-connected.

Theorem (Thomassen, 1980)
Every arc of a 3-strong tournament is contained in a Hamiltonian cycle.

Theorem (Thomassen, 1980)
Every strong tournament contains a vertex whose all out-arcs are contained in a Hamiltonian cycle.

Introduction

Theorem (Harary, Moser, 1966)

Every strong tournament is pancyclic.

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Theorem (Moon, 1966)
Every vertex of a strong tournament is pancyclic.

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Theorem (Moon, 1966)
Every vertex of a strong tournament is pancyclic.

Theorem (Alspach, 1967)
Every arc of a regular tournament is pancyclic.

Introduction

Theorem (Moon, 1994)

Every strong tournament has a Hamiltonian cycle containing at least 3 pan-cyclic arcs.

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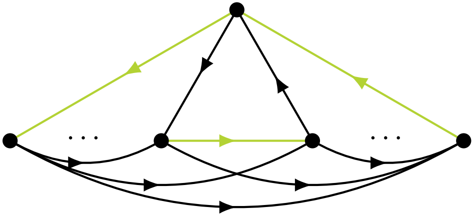

Introduction

Conjecture (Havet, 2004)

Every k-strong tournament has a Hamiltonian cycle containing at least $2k+1$ pancyclic arcs.

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Every k-strong tournament has a Hamiltonian cycle containing at least $2k+1$ pancyclic arcs.

Theorem (Havet, 2004)
Every 2-strong tournament has a Hamiltonian cycle containing at least 5 pancyclic arcs.

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Conjecture (Havet, 2004)

Every k-strong tournament has a Hamiltonian cycle containing at least $2k+1$ pancyclic arcs.

Theorem (Havet, 2004)

Every 2-strong tournament has a Hamiltonian cycle containing at least 5 pancyclic arcs.

Theorem (Yeo, 2005)

Every k-strong tournament has a Hamiltonian cycle containing at least $\frac{k+5}{2}$ pancyclic arcs.

Introduction

Theorem (Thomassen, 1980)

Every strong tournament contains a vertex whose all out-arcs are contained in a Hamiltonian cycle.

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Every strong tournament contains a vertex whose all out-arcs are contained in a Hamiltonian cycle.

Theorem (Yao, Guo, Zhang, 2000)

Every strong tournament contains a vertex whose all out-arcs are pancyclic.

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Theorem (Thomassen, 1980)

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Theorem (Yao, Guo, Zhang, 2000)

Every strong tournament contains a vertex whose all out-arcs are pancyclic.

Theorem (Guo, Li, Li, Zhao, 2014)

Every strong tournament with minimum out-degree at least 2 contains 3 vertices whose all out-arcs are pancyclic.

Introduction

Corollary (Guo, S, 2014)

Every Hamiltonian cycle of a tournament contains at least 3 pancyclic arcs.

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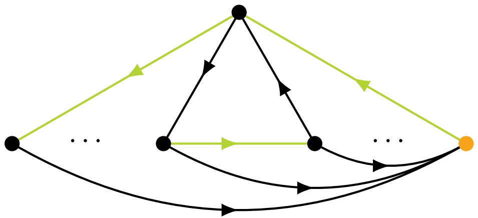
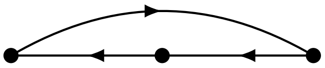


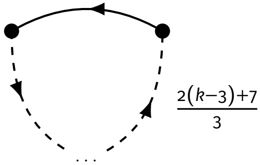
Improved bound

Theorem (2014)

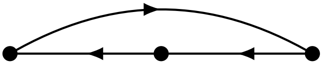
Every k-strong tournament has a Hamiltonian cycle containing at least $\frac{2k+7}{3}$ pancyclic arcs.

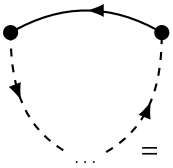
Proof (sketch)





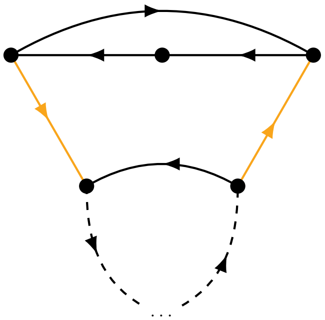
Proof (sketch)





$$\frac{2(k-3)+7}{3} + 2 = \frac{2k+7}{3}$$

Proof (sketch)



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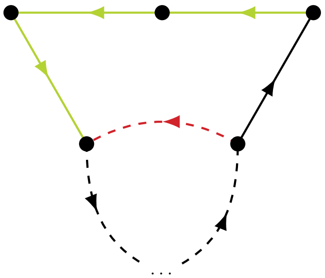

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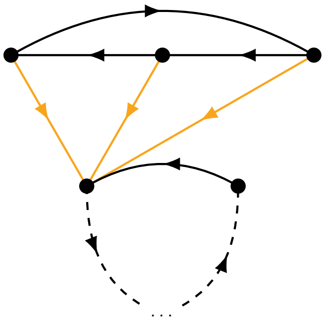
Proof (sketch)



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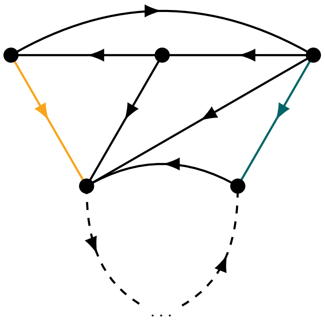

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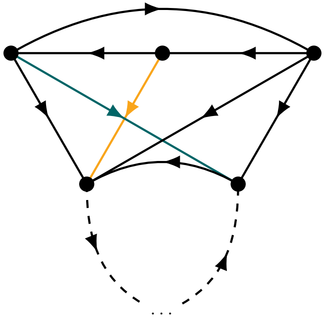

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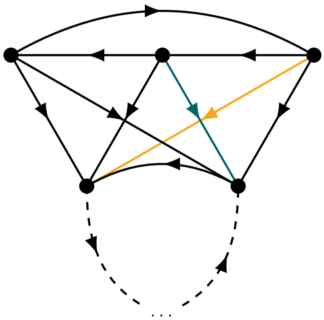

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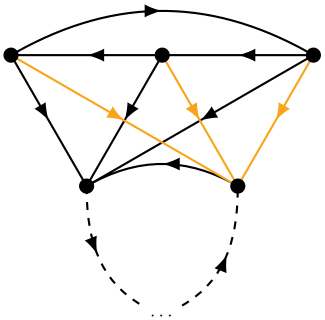

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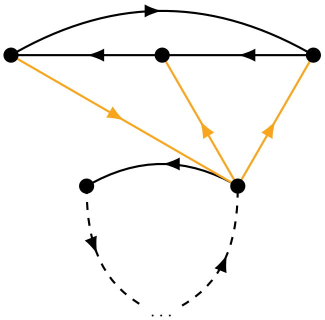
Proof (sketch)



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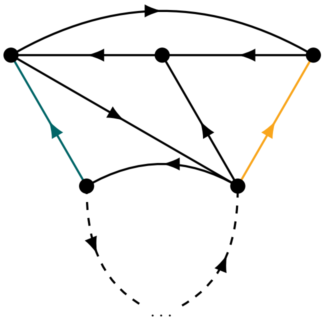

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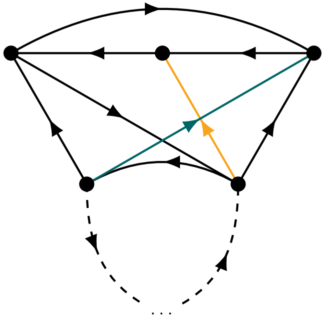
Proof (sketch)



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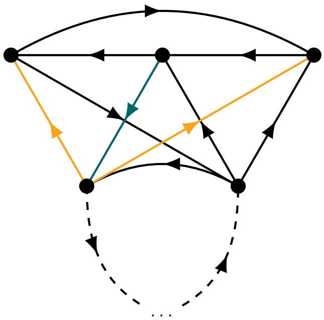

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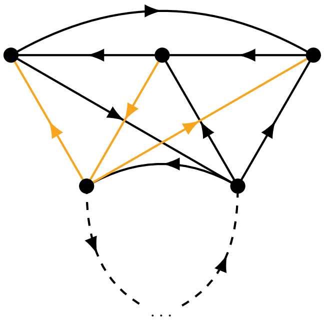

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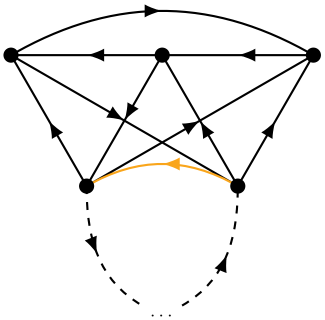

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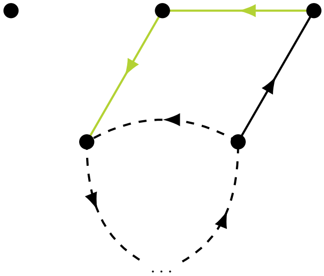

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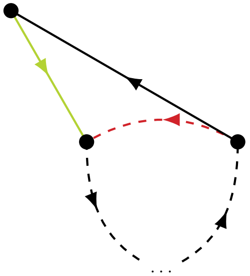

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Thank you.

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