Pancyclic arcs in Hamiltonian cycles of tournaments

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Introduction

Theorem (Rédei, 1934)

Every tournament contains a Hamiltonian path.
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Every tournament contains a Hamiltonian path.

Theorem (Camion, 1959)
Every strong tournament contains a Hamiltonian cycle.
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Theorem (Rédei, 1934)
Every tournament contains a Hamiltonian path.

Theorem (Camion, 1959)
Every strong tournament contains a Hamiltonian cycle.

Theorem (Thomassen, 1980)
For every $k \geq 1$ there are infinitely many tournaments with precisely $k$ Hamiltonian cycles.
Introduction

Theorem (Thomassen, 1980)
Every 4-strong tournament is strongly Hamiltonian-connected.
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Theorem (Thomassen, 1980)
Every arc of a 3-strong tournament is contained in a Hamiltonian cycle.
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Theorem (Thomassen, 1980)
Every 4-strong tournament is strongly Hamiltonian-connected.

Theorem (Thomassen, 1980)
Every arc of a 3-strong tournament is contained in a Hamiltonian cycle.

Theorem (Thomassen, 1980)
Every strong tournament contains a vertex whose all out-arcs are contained in a Hamiltonian cycle.
Introduction

Theorem (Harary, Moser, 1966)
Every strong tournament is pancyclic.
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Theorem (Harary, Moser, 1966)
Every strong tournament is pancyclic.

Theorem (Moon, 1966)
Every vertex of a strong tournament is pancyclic.
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Theorem (Harary, Moser, 1966)
Every strong tournament is pancyclic.

Theorem (Moon, 1966)
Every vertex of a strong tournament is pancyclic.

Theorem (Alspach, 1967)
Every arc of a regular tournament is pancyclic.
Introduction

Theorem (Moon, 1994)
Every strong tournament has a Hamiltonian cycle containing at least 3 pancyclic arcs.
Introduction

**Theorem (Moon, 1994)**

Every strong tournament has a Hamiltonian cycle containing at least 3 pancyclic arcs.
Introduction

Conjecture (Havet, 2004)
Every $k$-strong tournament has a Hamiltonian cycle containing at least $2k+1$ pancyclic arcs.
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Every $k$-strong tournament has a Hamiltonian cycle containing at least $2k+1$ pancyclic arcs.

Theorem (Havet, 2004)
Every 2-strong tournament has a Hamiltonian cycle containing at least 5 pancyclic arcs.
Introduction

Conjecture (Havet, 2004)
Every \( k \)-strong tournament has a Hamiltonian cycle containing at least \( 2k+1 \) pancyclic arcs.

Theorem (Havet, 2004)
Every 2-strong tournament has a Hamiltonian cycle containing at least 5 pancyclic arcs.

Theorem (Yeo, 2005)
Every \( k \)-strong tournament has a Hamiltonian cycle containing at least \( \frac{k+5}{2} \) pancyclic arcs.
Introduction

Theorem (Thomassen, 1980)
Every strong tournament contains a vertex whose all out-arcs are contained in a Hamiltonian cycle.
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Theorem (Thomassen, 1980)
Every strong tournament contains a vertex whose all out-arcs are contained in a Hamiltonian cycle.

Theorem (Yao, Guo, Zhang, 2000)
Every strong tournament contains a vertex whose all out-arcs are pancyclic.
Introduction

Theorem (Thomassen, 1980)
Every strong tournament contains a vertex whose all out-arcs are contained in a Hamiltonian cycle.

Theorem (Yao, Guo, Zhang, 2000)
Every strong tournament contains a vertex whose all out-arcs are pancyclic.

Theorem (Guo, Li, Li, Zhao, 2014)
Every strong tournament with minimum out-degree at least 2 contains 3 vertices whose all out-arcs are pancyclic.
Introduction

Corollary (Guo, S, 2014)
Every Hamiltonian cycle of a tournament contains at least 3 pancyclic arcs.
Introduction

Corollary (Guo, S, 2014)
Every Hamiltonian cycle of a tournament contains at least 3 pancyclic arcs.
Improved bound

Theorem (2014)

Every $k$-strong tournament has a Hamiltonian cycle containing at least $\frac{2k+7}{3}$ pancyclic arcs.
Proof (sketch)

\[
\frac{2(k-3) + 7}{3}
\]
Proof (sketch)

\[ 
\frac{2(k-3)+7}{3} + 2 = \frac{2k+7}{3} 
\]
Proof (sketch)
Proof (sketch)
Proof (sketch)

\[
\begin{align*}
(k - 3) + 2 &= k + 3 + 2 \\
(k - 3) + 2 &= k + 3 + 2 \cdot \cdots
\end{align*}
\]
Proof (sketch)

\[ (k - \frac{2}{3}) + \frac{7}{3} + \frac{1}{2} = k + \frac{7}{3} \]

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Proof (sketch)
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\[
(k - \frac{3}{7}) + \frac{2}{3} + \frac{2}{7} = \frac{2}{7} k + \frac{2}{3}
\]

\[
(k - \frac{3}{7}) + \frac{2}{3} + \frac{2}{7} = \frac{2}{7} k + \frac{2}{3}
\]

\[
\ldots
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Proof (sketch)
Proof (sketch)
Proof (sketch)
Proof (sketch)
References


Thank you.