ABSTRACTS

of the

GHENT GRAPH THEORY WORKSHOP

on

LONGEST PATHS AND LONGEST CYCLES

Degree-constrained spanning trees

Kathie Cameron
(Wilfrid Laurier University, Canada)

The *opposite tree problem* is: Given a graph G and a spanning tree T of G, is there another spanning tree, T' of G, such that for each vertex, its degree in T' is different from its degree in T? Such a tree T' is called an *opposite* of T. The existence of a Hamiltonian path in a graph is an instance of the opposite tree problem. It follows that the opposite tree problem is NP-complete in general. However, for some classes of graphs including complete graphs and complete bipartite graphs, we have characterized when an opposite tree exists and we show how to find an opposite tree when it exists.

I will also discuss two related NP-complete problems:

The same-degree tree problem: Given a graph G and a spanning tree T of G, is there another spanning tree, T', of G such that for each vertex, its degree in T' is the same as its degree in T? Such a tree T' is called a *same-degree tree* of T.

The intermediate tree problem: Given two distinct spanning trees, T and T', of a graph G, is there another spanning tree, T'', of G such that for each vertex, its degree in T'' is between its degree in T and its degree in T'? Such a tree T'' is called an *intermediate tree* of T and T'.

The same-degree tree problem includes the existence of a second Hamiltonian path with the same endpoints as a given Hamiltonian path. The intermediate tree problem includes the existence of a third Hamiltonian path given two Hamiltonian paths.

The work on opposite trees is joint with Kevin Kalanda, Bill Sands and Maria Sobchuk; same-degree trees is joint with Jack Edmonds; and intermediate trees is joint with Joanna Fawcett.

Finding minimal obstructions to graph colouring

JAN GOEDGEBEUR
(Universiteit Gent, Belgium)

For several graph classes without long induced paths there exists a finite forbidden subgraph characterisation for k-colorability. Such a finite set of minimal obstructions allows to provide a "no-certificate" which proves that a graph is not k-colorable.

In this talk we will present a new algorithm for generating all minimal forbidden subgraphs for k-colorability for given graph classes.

We will show how the new generation algorithm has been applied to fully characterise the forbidden subgraphs for k-colorability of various classes of graphs without long induced paths. Using this algorithm (combined with new theoretical results) we haven proven amongst others that there are 24 minimally non-3-colorable graphs in the class of P_6-free graphs, which solves an open problem posed by Golovach et al.

Joint work with Maria Chudnovsky, Oliver Schaudt and Mingxian Zhong.

On longest cycles in essentially 4-connected planar graphs

JOCHEN HARANT
(TU Ilmenau, Germany)

A planar 3-connected graph G is essentially 4-connected if, for any 3-separator S of G, one component of the graph obtained from G by removing S is a single vertex. Jackson and Wormald proved that an essentially 4-connected planar graph on n vertices contains a cycle C such that $|V(C)| \geq \frac{2n+4}{5}$. For a cubic essentially 4-connected planar graph G, Grünbaum with Malkevitch, and Zhang showed that G has a cycle on at least $\frac{3}{4}n$ vertices. In the present paper the result of Jackson and Wormald is improved. Moreover, new lower bounds on the length of a longest cycle of G are presented if G is an essentially 4-connected planar graph of maximum degree 4 or G is an essentially 4-connected maximal planar graph.

Barnette was right: not only fullerene graphs are Hamiltonian

František Kardoš
(Université Bordeaux 1, France)

Barnette conjectured in the 60s that all polyhedral (3-connected and planar) cubic graphs with faces of size at most six are Hamiltonian. The conjecture covers the case of the fullerene graphs, 3-connected planar cubic graphs with pentagonal and hexagonal faces only. We will present some of the ideas that led to a computer-assisted proof of the conjecture.

Complexity questions for minimally *t*-tough graphs

Gyula Y. Katona
(Budapest University of Technology and Economics, Hungary)

A graph G is minimally t-tough if the toughness of G is t and the deletion of any edge from G decreases the toughness. Kriesell conjectured that for every minimally 1-tough graph the minimum degree $\delta(G) = 2$. In the present talk we investigate different complexity question related to this conjecture. First we show that recognizing minimally t-tough graphs is a hard task for some t values, it is a DP-complete problem (implying that is probably even harder than being NP-hard.) Does this change if the question is for some special graph classes like chordal, split, claw-free and $2K_2$-free graphs and special t values? The answers vary. In some cases there are no such graphs at all, so it is really easy to recognize them. In some other cases, we can characterize all the graphs. Yet in some case we can at least recognize the in polynomial time.

Many open questions remain.

Joint work with Kitti Varga and István Kovács.

Long properly colored cycles in edge-colored complete graphs

Ruonan Li
(Northwestern Polytechnical University, P. R. China and Universiteit Twente, Netherlands)

We study the existence of properly edge-colored cycles in (not necessarily properly) edge-colored complete graphs. Fujita and Magnant [S. Fujita and C. Magnant, Properly colored paths and cycles, *Discrete Appl. Math.* **159** (2011) 1391–1397] conjectured that in an edge-colored complete graph on n vertices with minimum color degree at least $(n+1)/2$, each vertex is contained in a properly edge-colored cycle of length k, for all k with $3 \leq k \leq n$. They confirmed the conjecture for $k = 3$ and $k = 4$, and they showed that each vertex is contained in a properly edge-colored cycle of length at least 5 when $n \geq 13$, but even the existence of properly edge-colored Hamilton cycles is unknown (in complete graphs that satisfy the conditions of the conjecture). We prove a cycle extension result that implies that each vertex is contained in a properly edge-colored cycle of length at least the minimum color degree, i.e., the smallest number of different colors appearing on the edges incident with the vertices of the graph.

Joint work with Hajo Broersma, Chuandong Xu and Shenggui Zhang.

A streaming algorithm for the undirected longest path problem

Lasse Kliemann
(Kiel University, Germany)

We present the first streaming algorithm for the longest path problem in undirected graphs. The input graph is given as a stream of edges and RAM is limited to only a linear number of edges at a time (linear in the number of vertices n). We prove a per-edge processing time of $O(n)$, where a naive solution would have required $\Omega(n^2)$. Moreover, we give a concrete linear upper bound on the number of bits of RAM that are required.

On a set of graphs with various structure, we experimentally compare our algorithm with three leading RAM algorithms: Warnsdorf (1823), Pohl-Warnsdorf (1967), and Pongracz (2012). Although conducting only a small constant number of passes over the input, our algorithm delivers competitive results: after removing the 10% worst cases, we obtain at least 70% of the best solution returned by any RAM algorithm. On many graph classes, we obtain that quality even in the worst case. On some graph classes, we deliver better results than any of the RAM algorithms.

Joint work with Christian Schielke and Anand Srivastav.

Hamiltonicity of graphs on surfaces

Kenta Ozeki
(National Institute of Informatics, Japan)

Tutte showed that "every 4-connected planar graph is Hamiltonian", and Thomassen extended it, showing that "every 4-connected planar graph is Hamiltonian-connected", i.e., there is a Hamiltonian path connecting any two prescribed vertices. From those results, several improvements have been shown: for example, Hamiltonicity of graphs on non-spherical surfaces. In this talk, I will give a survey on recent results, and I also would like to give a basic strategy to prove them.

Partially joint work with Ken-ichi Kawarabayashi.

Edge colorings and circular flow numbers of regular graphs

Eckhard Steffen
(Paderborn University, Germany)

A graph G is a class 1 graph if $\chi'(G) = \Delta(G)$ and it is a class 2 graph if $\chi'(G) > \Delta(G)$, where $\chi'(G)$ is the edge-chromatic number of G. The circular flow number of G is $\inf\{r|G$ has a nowhere-zero r-flow$\}$, and it is denoted by $F_c(G)$. It is known that $F_c(G)$ is always a minimum and that it is a rational number.

Furthermore, if r is the circular flow number of a $(2t+1)$-regular graph, then either $r = 2 + \frac{1}{t}$ or $r \geq 2 + \frac{2}{2t-1}$.

We characterize $(2t+1)$-regular graphs with circular flow number $2 + \frac{2}{2t-1}$. For $t = 1$ this is Tutte's characterization of cubic graphs with circular flow number 4. The class of cubic graphs is the only class of odd regular graphs where a flow number separates the

class 1 graphs from the class 2 graphs. However, our results imply that a $(2t+1)$-regular graph G with $F_c(G) \leq 2 + \frac{2}{2t-1}$ is a class 1 graph.

We study the question whether there are $(2t+1)$-regular graphs H_1 and H_2 such that H_1 is class 1, H_2 is class 2, and $F_c(H_1) = F_c(H_2) = r > 2 + \frac{2}{2t-1}$. We give an affirmative answer to this question for some specific values of r.

We finally state some conjectures and relate them to Jaeger's circular flow conjecture and Tutte's 3-flow conjecture.

Pancyclic arcs in Hamiltonian cycles of tournaments

Michel Surmacs
(RWTH Aachen, Germany)

In one of the earliest and most basic results on this class of directed graphs, Camion proved that the longest cycle of a strongly connected tournament is Hamiltonian. This result was generalized to what is now well-known as Moon's theorem, which states that every vertex of a strongly connected tournament of order n is pancyclic, i.e. contained in a cycle of length ℓ for all $\ell \in \{3, \ldots, n\}$. Alspach showed that the same is true for the arcs of regular tournaments, but not for the arcs of strongly connected tournaments in general. Moon then gave a lower bound for the number of pancyclic arcs guaranteed to be contained in a strongly connected tournament. In fact, he proved the existence of a Hamiltonian cycle containing at least three pancyclic arcs in every strongly connected tournament. As Moon also found and characterized tournaments for which this bound is sharp, Havet considered 2-strong tournaments to find a better one. His lower bound of at least 5 pancyclic arcs contained in a Hamiltonian cycle in 2-strong tournaments was generalized by Yeo to $\frac{k+5}{2}$ in k-strong tournaments, $k \geq 1$. We improve his bound to $\frac{2k+7}{3}$ for k-strong tournaments, $k \geq 1$.

Chords in longest cycles

Carsten Thomassen
(Technical University of Denmark)

In 1976 I made the conjecture that every longest cycle in every 3-connected graph has a chord. Around the same time, Andrew Thomason introduced his elegant lollipop method to prove, among other things, that every r-regular Hamiltonian graph has a second Hamiltonian cycle when r is odd.

Sheehan's conjecture says that this also holds for r even.

In this talk I discuss the connections between these results and conjectures, and I describe some recent progress on the chord conjecture.

Connections between decomposition trees of 3-connected plane triangulations and Hamiltonian properties

Nico Van Cleemput
(Universiteit Gent, Belgium)

We investigate Hamiltonian properties of plane triangulations. The central part of the talk is the search for the strongest possible form of Whitney's theorem about Hamiltonian triangulations in terms of the decomposition tree defined by separating triangles. Jackson and Yu showed that a triangulation is Hamiltonian if this decomposition tree has maximum degree 3. We will decide on the existence of non-Hamiltonian triangulations with given decomposition trees for all trees except trees with exactly one vertex with degree $k \in \{4, 5\}$ and all other degrees at most 3. For these cases we show that it is sufficient to decide on the existence of non-Hamiltonian triangulations with decomposition tree $K_{1,4}$ or $K_{1,5}$. These results were obtained using a combination of computational results and theoretical results. We also focus on stronger and weaker variants of Hamiltonicity and show what can be or cannot be decided based upon the decomposition tree.

Joint work with Gunnar Brinkmann and Jasper Souffriau.

On the minimum degree of minimally 1-tough graphs

Kitti Varga
(Budapest University of Technology and Economics, Hungary)

Let $\omega(G)$ denote the number of components of a graph G. A graph G is called t-tough for a positive real number t, if $\omega(G-S) \leq |S|/t$ for any cutset S of G. A graph G is said to be minimally t-tough, if it is t-tough, but removing any of its edges the resulting graph is no longer t-tough. Mader proved that every minimally k-connected graph has a vertex of degree k. Kriesell conjectured an analog of Mader's theorem: every minimally 1-tough graph has a vertex of degree two. First, we show that the family of minimally t-tough graphs is rich: any graph can be embedded as an induced subgraph into a minimally t-tough graph. Our main result is that every minimally 1-tough graph of order n has a vertex of degree at most $n/3+3$. We also examine the conjecture in a special case. Matthews and Sumner proved that a noncomplete claw-free graph is $2t$-connected if and only if it is t-tough. Using this theorem we show that minimally 1-tough claw-free graphs are cycles, which implies that in this graph family the conjecture is trivially true.

Joint work with Gyula Y. Katona and Dániel Soltész.

Finding spanning trees with few leaves using DFS

Gábor Wiener
(Budapest University of Technology and Economics, Hungary)

Finding spanning trees of a connected graph with as few leaves as possible is an obvious generalization of the problem of traceability. We show how different versions of depth first search can be used to find spanning trees with few leaves in certain graph classes. E.g. we give a common generalization of a theorem of Kano et al. and Ainouche et al. concerning claw-free graphs.