Visualizing Cauchy's Interlacing Property for Line Distance Matrices

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Let $\boldsymbol{t} = (t_1, t_2, \ldots, t_n)$, $0 < t_1 < t_2 < \cdots < t_n$, $t_i \in \mathbb{R}$, be a given vector. A **line distance matrix** $A \in \mathbb{R}^{n\times n}$ is defined as

$$(A)_{ij} = \begin{cases} t_i - t_j, & j \le i, \\ t_j - t_i, & i < j. \end{cases}$$

Line distance matrices frequently appear in graph theory and in bioinformatics. Consider a DNA sequence (of four nucleotide A,T,G,C) and represent distances between occurrences of G. For example, the first exon of human b-globin gene starts as: ATGGTGCACCTGACTCCTGAG... Positions of G in the sequence can be written in a vector $\boldsymbol{t} = (3, 4, 6, 12, 19, 21)$ and its corresponding line distance matrix is

$$A = \begin{pmatrix} 0 & 1 & 3 & 9 & 16 & 18 \\ 1 & 0 & 2 & 8 & 15 & 17 \\ 3 & 2 & 0 & 6 & 13 & 15 \\ 9 & 8 & 6 & 0 & 7 & 9 \\ 16 & 15 & 13 & 7 & 0 & 2 \\ 18 & 17 & 15 & 9 & 2 & 0 \end{pmatrix}. \tag{1}$$

In the paper the distribution of the eigenvalues of line distance matrices is studied. It is shown that line distance matrices of size n have one positive and $n-1$ negative eigenvalues. Visual representation of Cauchy's interlacing property for line distance matrices is considered.

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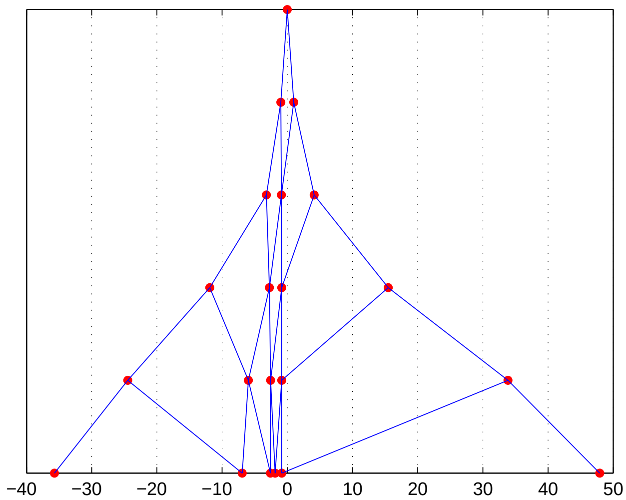

Figure 1: Cauchy's interlacing property for line distance matrix (1).