Communication network for the GPS III system

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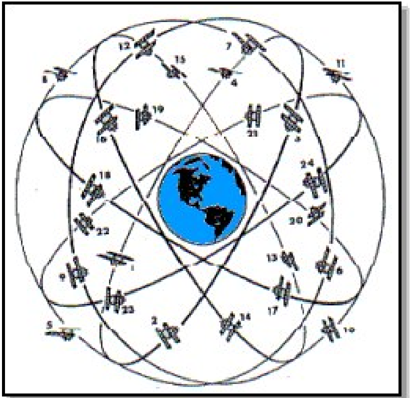
July 2010

1. Introduction
2. First test
3. Final program
4. Results
5. Conclusion




Advantages of communication

- continuous telemetry
- frequent updates

Restrictions for connections

- connection possible during the entire orbit
- 4 connections a satellite

Definition

V_{GPS} is the set of all satellites.

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$G_{Pos} = (V_{GPS}, E_{Pos})$, *with* $\forall v, w \in V_{GPS} : \{v, w\} \in E_{Pos} \Leftrightarrow$ *communication between satellites* v *and* w *is technically possible.*

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Definition

*A **connection graph** is a spanning, 4-regular subgraph of G_{Pos}.*

Definition

Given a graph $G = (V, E)$ *and a vertex* $v \in V$: *the* **neighbourhood** *of* v *in* G *is* $N(v, G) = \{w \in V | \{v, w\} \in E\}$.

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Definition

The function **uredop** *of a vertex v with respect to its neighbours is defined as:*

Let $M_4 = \{M \subseteq V_{GPS} || M| = 4\}$.

$uredop : V_{GPS} \times M_4 \rightarrow \mathbb{R}^+ \bigcup \{\infty\}$, so that

$\forall v \in V_{GPS} : M \not\subset N(v, G_{Pos}) \Rightarrow uredop(v, M) = \infty$.

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Definition

The uredop value of a vertex v in a connection graph G is defined as:
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The uredop value of a connection graph G is defined as: $uredop(G) = max\{uredop(v, G) | v \in V_{GPS}\}$.

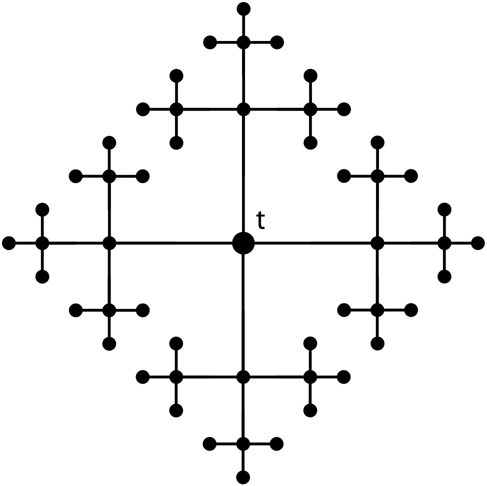
Minimum requirements

Construct a connection graph G which

- is 4-regular
- is a subgraph of G_{Pos}
- has diameter at most 4
- has $uredop(G) < 3$

Best connection graph G

- Diameter 3
- Smallest possible value for $uredop(G)$
- Maximum diameter 4 after one edge removal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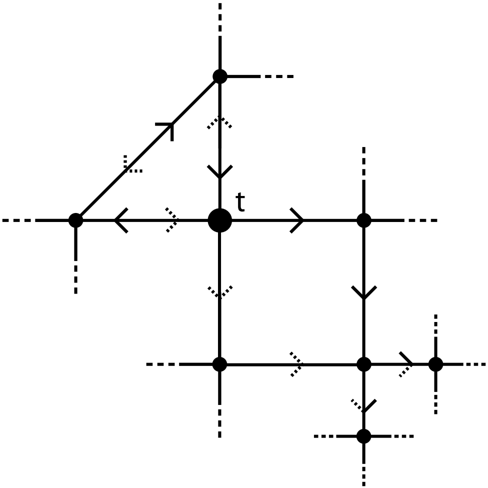
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Questions?

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Questions

- Are there connection graphs with diameter 3?
- How many?
- With which uredop value?

Method

- Generate 4-regular graphs
- Filter graphs with diameter 3
- Determine whether they are subgraph of G_{Pos}
- Determine uredop value

Results

- Thousands of millions of 4-regular graphs with 27 vertices and diameter 3
- Almost all tested graphs were subgraph of G_{Pos}
- Very different uredop values (also good ones)

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Conclusion

The uredop values will have to provide the biggest restriction.

Definition

Given a graph $G = (V, E)$*. A vertex set* $IS \subseteq V$ *is an* **independent set** *in* $G \Leftrightarrow \forall v, w \in IS : \{v, w\} \notin E$.

Definition

Given a graph $G = (V, E)$*. A vertex set* $C \subseteq V$ *is a* **clique** *in* $G \Leftrightarrow \forall v, w \in C : \{v, w\} \in E$.
If there doesn't exist a clique C' *with* $|C'| > |C|$*, then* C *is a maximum clique in* G.

Definition

$G_C(u) = (V_C(u), E_C(u))$, *with*
$V_C(u) = \{(s, N) | s \in V_{GPS}, N \subset N(s, G_{Pos}), |N| = 4$ *and*
$uredop(s, N) < u\}$ *and*
$E_C(u) = \{\{v, w\} | v = (s, N_s) \in V_C(u), w = (t, N_t) \in V_C(u), s \neq t$ *and*
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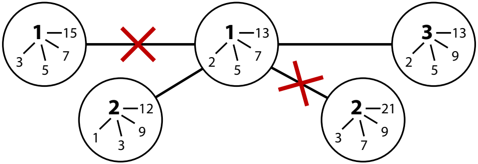

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Theorem

The largest cliques in $G_C(u)$ have at most 27 vertices.

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Every connection graph $G = (V_{GPS}, E)$ with uredop values smaller than u, corresponds to a maximum clique C in $G_C(u)$, with $|C| = 27$.

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Theorem

Every maximum clique C in $G_C(u)$, with $|C| = 27$, corresponds to a connection graph with uredop smaller than u.

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Definition

The language **Uredop** *is the set of all strings with the structure* $g'\#c\#u$, *with*

- g' *represents a graph* G'
- c *represents all possible sets of 4 neighbours with corresponding value for each vertex*
- u *represents a natural number* U

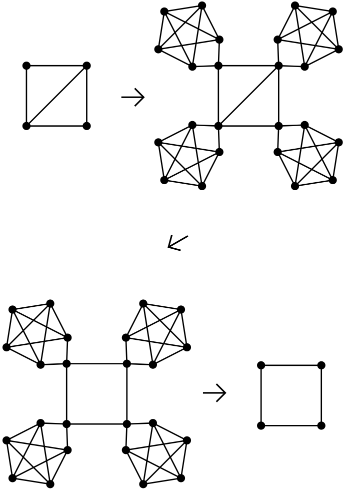
and for which there exists a subgraph G *of* G', *such that* $C(G) < U$.

Theorem

The language Uredop is NP-complete.

Proof

It is easy to see that Uredop $\in$ *NP.*
We still have to prove that each problem in NP can be reduced to Uredop in polynomial time.
To prove this, we reduce the language HC (graphs containing a Hamiltonian cycle) to Uredop.



General clique solver

Too slow for this specific problem

Pseudocode recursion

- Determine central vertex with smallest number of possible neighbourhoods
- Amount = 0: backtrack
- For each possible neighbourhood:
  - choose neighbourhood
  - adjust lists with possible neighbourhoods
  - found connection graph or continue in recursion

Some details

- Diameter as bounding criterium
- Use of bitvectors
- Reduce iteration length
- Efficient adjustment of lists with possible neighbourhoods

Adjust list with possible neighbourhoods

- Split lists once every recursion step
- Only adjust lists of (not yet visited) neighbours G_{Pos}
- Pass the corresponding lists

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Split with and without 2

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1	4	7	13	19
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Programs

- Independent set method
- Independent set method (Gunnar Brinkmann)
- General clique solver (Patric Östergård)

Determine correctness

- Compare some specific graphs
- Compare the number of graphs per diameter, given an upper bound for uredop

Best graphs

Restrictions	Diameter (D)	Diameter after edge removal (E)	uredop value
D 4	4	5	0.92423
D 4, E 4	4	4	0.93317
D 3	3	5	0.96598
D 3, E 4	3	4	0.97854

For our practical problem

- Best graphs within the hour
- Much better results than those found by Lockheed Martin

Time needed depends heavily on the input weights.

Possible variations of input:

- Different amount of vertices
- Different values weight function
- Different amount of connections per vertex
- Different G_{Pos}

Thank you!