An algorithm for determining the most stable vacancy clusters in diamond lattice

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# - Introduction

- Vacancy clusters in silicon and diamond
- Algorithm for construction of vacancy clusters in the diamond structure of carbon
  - Results
  - Conclusions

# Introduction

In diamond more than 500 electronic and more than 150 vibrational optical centers have been documented Many of them are due to  $V_n$  vacancy centers.

Vacancy clusters in diamond and in silicon are detected by electron paramagnetic resonance, positron annihilation spectroscopy and other methods.

Usually they are produced by electron, neutron, or ion irradiations and by temperature annealing.

### Vacancy: mono vacancy

Vacancy cluster: connected set of mono vacancies

 $V_n$ : vacancy cluster of n mono vacancies

Vacancies and vacancy clusters will be represented by the missing atoms from the bulk



Fig. 1. Isochronal annealing-in and annealing-out of EPR spectra [4,7].

#### J. M. Baker, Diam. and Rel. Mater. 16 (2007) 216-219

# Representation of a $V_6$ vacancy cluster





K. Iakoubovskii and A. Stesmans Phys. Stat. Sol (a) 201. (2004) 2509-2515





(e) V<sub>9</sub> (R7a) C<sub>2v</sub>

(f) V<sub>11</sub> (R7) C<sub>1h</sub>

(g) V13 (R8) C2v





(h) V7 (R7a) C2

(i) V<sub>8</sub> D<sub>2d</sub>

(j) V<sub>7</sub> (R7) C<sub>1h</sub>

J. M. Baker, Diam. and Rel. Mater. 16 (2007) 216-219

Based on the counting of dangling bonds, it has been proposed that closed ring structures of vacancies  $V_6$  and  $V_{10}$  should be especially stable in silicon.



(D.J. Chadi and K.J. Chang, Phys. Rev. B38, 1523, (1988).)

Adamantane like vacancy clusters:

Vacancy cluster constructed by minimizing the number of dangling bonds in the vacancy cluster

# Adamantane like vacancy clusters from $V_2$ to $V_{14}$ in silicon.



J. L. Hastings et al., Phys. Rev. B56, 10215 (1997)
A. Bongiorno et al. Europhysics Letters 59, 608 (2000)
T.E.M. Staab et al., Phys. Rev. B65, 115210 (2002)

# Adamantane like vacancy clusters from $V_{15}$ to $V_{18}$ in silicon.



A. Bongiorno et al. Europhysics Letters 59, 608 (2000) T.E.M. Staab et al., Phys. Rev. B65, 115210 (2002)



L. S. Hounsome et al. Phys. Stat. Sol (a) 202. (2005) 2182-2187

# Our goal is to:

- a. Enumerate all distinct structures of  $V_n$  vacancy clusters with increasing n.
- b. Evaluate a large number of  $V_n$  vacancy clusters at a realistic level of quantum mechanics
- c. Interpret the driving forces of the distortions.

I. Laszlo, M. Kertesz, B. Slepetz, Y. Gogotsi Diamond Relat. Mater. (2010), doi:10.1016/j.diamond.2010.05.001

# The method

-Super cell of N=216 atoms in diamond structure

- -The  $V_n$  vacancy is represented by taking away the  $V_n$  atomic cluster from the super cell
- Periodic boundary conditionTBDFT for the interactions

D. Porezag et al. Phys. Rev B51 (1995) 12947



-Conjugate gradient method for minimizing the  $E^{n}_{vac}$ total energy of the system of (216-n) atoms. (-1 < n < 15)

I. Laszlo, M. Kertesz, B. Slepetz, Y. Gogot Diamond Relat. Mater. (2010), doi:10.1016/j.diamond.2010.05.001 Relative stability of n-vacancy cluster geometries Formation energy

$$E_{F}^{n} = E_{vac}^{n} - \frac{N-n}{N} E_{cryst}^{N}$$

 $E_{\rm F}^{\rm n}$  Formation energy of n-vacancy cluster

 $E_{vac}^n$  Total energy of super cell with N-n atoms

$$E_{\text{cryst}}^{N} = E_{\text{vac}}^{0}$$

$$E_{\text{FV}}^{n} = \frac{E_{\text{F}}^{n}}{n}$$
Formation energy per vacancy

# Algorithm for the construction a diamond vacancy clusters



# Selection of equivalent structures





Diagonalization of the modified adjacency matrix  $D_{ij}=exp(-ar_{ij})$  of the corresponding complete graph.

 $r_{ij}$  is the Euclidean distance in the diamond lattice Between vertices i and j. a = 1.0 Angstrom The number of all possible  $V_n$  vacancy clusters n : number of vacancies

- p : number of generated vacancy clusters
- q : number of in equivalent vacancy clusters





n	p	q	
1	1	1	
2	4	1	
3	6	1	
4	8	3	
5	30	7	
6	83	24	
7	328	88	
8	1357	385	
9	6617	1713	
10	32417	8112	
11	167511	38865	
12	869139	190081	
13	4574468	937194	
14	24139560	4660000	I. Laszlo, M. Kertesz, B. Slepetz, Y. Gogo Diamond Relat. Mater. (2010), doi:10.1016/j.diamond.2010.05.001

Algorithm for generating connected vacancy clusters

- 1. Start with  $V_1$  and increase n one by one
- 2. Generate all possible  $V_n$  from  $V_{n-1}$
- 3. Eliminate the equivalent vacancy clusters.  $I_n$  is the number of in-equivalent structures
- 4. Optimize the geometries of all  $I_n$  structures
- 5. Calculation of formation energies for all  $V_n$
- 6. Keep only the M<sub>n</sub> lowest energy vacancy clusters
  7.n=n+1 and GO TO 2. (The process terminates at a predetermined value n.)

I. Laszlo, M. Kertesz, B. Slepetz, Y. Gogotsi Diamond Relat. Mater. (2010), doi:10.1016/j.diamond.2010.05.001 Algorithm for generating connected vacancy clusters

Up to n=7, we included all possible vacancy clusters, for n > 7 we used the following parameters

$$M_7 = M_8 = M_9 = M_{10} = M_{11} = 5$$
 and

 $M_{12} = M_{13} = 7$ 

The number of all possible  $V_n$  vacancy clusters n : number of vacancies

- p : number of generated vacancy clusters
- q : number of in equivalent vacancy clusters



List of  $V_{4_L}$  parent structures for  $V_{5_k}$  structures

# SN serial number SNP serial number of parent structures

		V <sub>5</sub>				
SN		SNP				
1 2 3 4 5 6 7	1 1 1 1 3 2	2 3 2	3			

$V_{5_k}$	SN = k
$V_{4\_L}$	SNP = L

		Vé	
SN		SNP	
1	1		
2	1	5	
3	1	3	
4	1	3	4
5	1	2	5
6	1	4	
7	1	2	
8	2	3	
9	2	4	
10	2	3	4
11	3		
12	3	6	
13	3		
14	4	6	
15	3	4	5
16	4		
17	2	4	5
18	3	4	
19	3	4	6
20	2	4	7
21	3		
22	3	5	
23	5		
24	6		

		V <sub>7</sub>		
SN		SNP		
1	1	3	4	6
2	1	2	5	7
3	2	5	7	
4	2	6	15	
5	2	3		
6	2			
7	3	7	8	
8	2	3	15	
9	2	22		
10	2			
11	7			
12	3	11		
13	4	5	15	20
14	4	5	8	15
15	5			
16	3	12		
17	3	6	19	
18	2	23		
19	4	12	14	
20	3	13		
21	2	4	22	
22	2	5	23	
23	3	4	21	
24	3	4	18	
25	2	4	17	

- 147	-		- 160 C	
27	5	10	22	
28	3	5	22	
29	2	5	17	
30	3	5	10	
31	14	20		
32	4	19		
33	6	14	18	
34	6	7	20	
35	4	10	13	
36	10	18	20	
37	12			
38	6	10		
39	8	12		
40	12			
41	7	10	17	
42	17	20		
43	5	17	23	
44	9	16	2.0	
45	4	7	9	10
46	4	6	16	18
47	8	11	13	
48	11	22		
49	10	11	15	
50	12	14	15	

51	5	6	9	17
52	5	7	9	17
53	8	9	10	18
54	15	17	23	
55	9	15	17	18
56	15	18	22	
57	12	24		
58	8	10	21	
59	13	21		
60	10	12	19	
61	8	19	20	
62	11	21		
63	10	17	18	
64	8	17	22	
65	11	12	19	
66	13	22		
67	17	19	22	
68	14	19	24	
69	12	13	19	
70	10	15	16	17
71	3			
72	9	10	14	19
73	23			
74	18	19	21	
75	18			
76	12	19	24	
77	24			
78	13	15	18	
79	12	18	19	
80	12	21		
81	12	22		
82	15	21	22	
83	22			
84	14	16	18	19
85	15	22	23	
86	14	17		
87	15	19		
88	22	23		









# Representation of a $V_6$ vacancy cluster

















Coulson and Kearsley,

Proc. Roy. Soc. Ser. A241 (1957) 433





T<sub>d</sub>



Td

V<sub>2\_1</sub> (6,4,4)





 $C_{2h} R4/W6^{20,35}$ 

V<sub>3\_1</sub> (8,5,5) 7 7 90000 7 90000

430b <sub>2</sub> <sup>0</sup>		$430b_2^0$
429a <sub>1</sub> °		429a <sub>1</sub> 0
428b1 <sup>0</sup>		428b1 <sup>0</sup>
427a2 <sup>0</sup>		427a2 <sup>0</sup>
426b <sub>2</sub> <sup>2</sup>	<u> </u>	426b <sub>2</sub> <sup>2</sup>
	$\equiv$	
422a <sub>1</sub> 2 =	 	422a <sub>1</sub> 2
421a <sub>1</sub> 2		421a <sub>1</sub> 2
420b <sub>1</sub> <sup>2</sup>		420b1 <sup>2</sup>





V<sub>3\_1</sub>





 $C_{2v} R 5^{2,6}$ 

### V<sub>4\_1</sub> (10,6,6)

7 5 5350 7 50555

429b <sup>0</sup>	429b <sup>0</sup>
428a <sup>0</sup>	428a <sup>0</sup>
427a <sup>0</sup>	427a <sup>0</sup>
426b <sup>0</sup>	426b <sup>0</sup>
425b <sup>0</sup>	425b <sup>0</sup>
424a <sup>2</sup>	424a <sup>2</sup>
=	
420b <sup>2</sup>	420b <sup>2</sup>
419a <sup>2</sup> ===	419a <sup>2</sup>
418b <sup>2</sup>	$418b^{2}$
417a <sup>2</sup>	417a <sup>2</sup>
$\wedge$	A
$C_2$	X
XSK I X	XII

 $V_{4_1}$ 





C<sub>2</sub>

	V <sub>5_1</sub> (12,6,7)	400.30
428a <sup>390</sup> 427a <sup>90</sup> 426a <sup>90</sup> 425a <sup>90</sup> 424a <sup>90</sup> 423a <sup>90</sup>		429a <sup>10</sup> 428a <sup>30</sup> 427a <sup>30</sup> 426a <sup>30</sup> 425a <sup>30</sup> 424a <sup>30</sup> 423a <sup>30</sup>
422a <sup>2</sup> 421a <sup>2</sup> 418a <sup>2</sup> 417a <sup>2</sup> 416a <sup>2</sup> 415a <sup>2</sup>		422a <sup>°2</sup> 418a <sup>°2</sup> 417a <sup>°°2</sup> 416a <sup>°°2</sup> 415a <sup>°°2</sup>



 $C_8 R 8^6$ 





 $C_{3v}$ 

 $V_{4_2}$ (10,6,6) 2027-00-0-000-000-000-00





 $C_{3v}$ 

#### $V_{4_1}$ (10,6,6) 429b<sup>0</sup> 429b<sup>0</sup> 428a<sup>0</sup> 428a<sup>0</sup> 427a<sup>0</sup> 427a<sup>0</sup> 426b<sup>0</sup> 426b<sup>0</sup> 425b<sup>0</sup> 425b<sup>0</sup> 424a<sup>2</sup>. 424a<sup>2</sup> 420b<sup>2-</sup>-420b<sup>2</sup> 419a<sup>2</sup>==== 419a<sup>2</sup> 418b<sup>2</sup> 418b<sup>2</sup> 417a<sup>2</sup> 417a<sup>2</sup>

C<sub>2</sub>







 $C_2$ 





 $D_{3d}$ 

#### V<sub>14\_1</sub> (20,10,18





			$V_1$			
SN	SNP	Х	Y	Ζ	Ef	$E_{fV}$
1		0	0	2	7.2720	7.2720
			$V_2$			
SN	SNP	Х	Y	Z	Ef	E <sub>fV</sub>
1	1	1	-1	1	10.4547	5.2274
			V3			
SN	SNP	Х	Y	Ζ	Ef	$E_{\texttt{fV}}$
1	1	0	-2	0	13.8462	4.6154

			₩4				
SN	SNP	X	Y	Ζ	Ef	EfV	
1 2 3	1 1 1	-1 2 1	-3 0 -3	1 0 -1	16.4986 17.1631 17.2365	4.1246 4.2908 4.3091	

			v 5				
SN	SNP	Х	Y	Z	Ef	EfV	
1	1	-2	-2	2	18.9534	3.7907	
3	3	-1	-1	-1	19.8946	3.9789	
4 5	3 1	-1 0	-3 -4	1 2	19.9947 20.0330	3.9989 4.0066	
6 7	3 2	0 2	-4 -2	-2 2	20.5336 20.5732	4.1067 4.1146	

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 $\nabla T \Sigma$ 

# Conclusions

- -The adamantane like structures do not describe the vacancies in the diamond structure of carbon
- -The tendency of local graphitization stabilizes the surface of diamond vacancy clusters.
- -Each tetrahedron of graphitization produced an extra energy level in the gap.
- -We described all possible vacancy clusters up to  $V_7$ .
- -Using five extra integers we described the structure of each voids.
- -There is a tendency for having graphite like vacancy surface