Crosscovers

Aleksander Malnič
University of Ljubljana

Joint work with Steve Wilson

Computers in Scientific Discovery 5
Sheffield, UK
July, 2010

Crosscovers: special covers (regular, irregular)

Crosscovers: special covers (regular, irregular)

X = connected graph, Γ = abelian group. Crossvoltage assignments

Crosscovers: special covers (regular, irregular)

X = connected graph, Γ = abelian group. Crossvoltage assignments

- $\zeta\colon E(X) \to \Gamma$, $e \mapsto \zeta_e$ **crossvoltages** assigned to unoriented edges

Crosscovers: special covers (regular, irregular)

X = connected graph, Γ = abelian group. Crossvoltage assignments

- $\zeta \colon E(X) \to \Gamma$, $e \mapsto \zeta_e$ **crossvoltages** assigned to unoriented edges

Derived crosscover $\mathrm{Cr}(X, \zeta)$

Crosscovers: special covers (regular, irregular)

X = connected graph, Γ = abelian group. Crossvoltage assignments

- $\zeta\colon E(X) \to \Gamma$, $e \mapsto \zeta_e$ **crossvoltages** assigned to unoriented edges

Derived crosscover $\mathrm{Cr}(X, \zeta)$

- vertices = $V(X) \times \Gamma$, edges = $E(X) \times \Gamma$

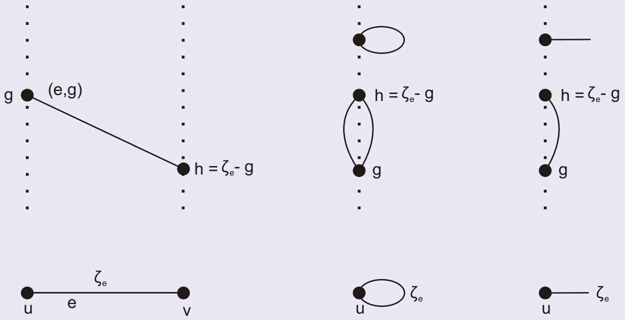
Crosscovers: special covers (regular, irregular)

X = connected graph, Γ = abelian group. Crossvoltage assignments

- $\zeta\colon E(X) \to \Gamma$, $e \mapsto \zeta_e$ **crossvoltages** assigned to unoriented edges

Derived crosscover $\mathrm{Cr}(X, \zeta)$

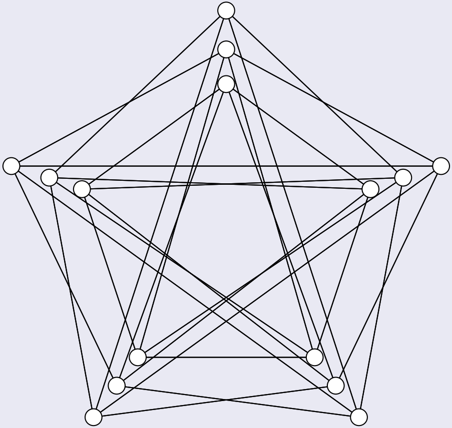
- vertices = $V(X) \times \Gamma$, edges = $E(X) \times \Gamma$
- (e, g) connects (u, g) and $(v, \zeta_e - g)$



Example: Line graph of the Petersen graph

Example: Line graph of the Petersen graph

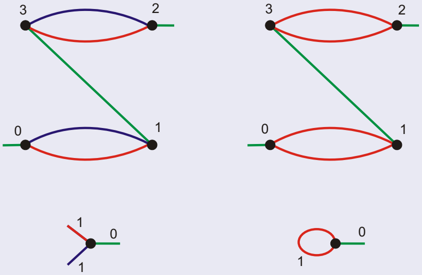
A $\mathbb{Z}_3$-crosscover of K_5



Example: Cayley crosscovers

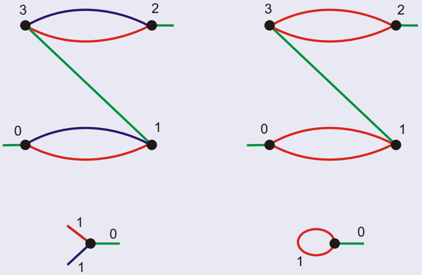
Example: Cayley crosscovers

Crosscovers of monopoles



Example: Cayley crosscovers

Crosscovers of monopoles



Cayley sum graphs

A subclass of Cayley crosscovers: $g, h \in \Gamma$ adjacent iff $g + h \in S$.

Motivation

Motivation

Stability of graphs

Motivation

Stability of graphs

- S. Wilson, Unexpected symmetries of graphs, JCTB '08

Motivation

Stability of graphs

- S. Wilson, Unexpected symmetries of graphs, JCTB '08
- $\mathrm{Aut}(X)$ lifts to $\mathrm{CDC}(X)$ as $\mathbb{Z}_2 \times \mathrm{Aut}(X)$. Wilson determined conditions when $\mathrm{Aut}(\mathrm{CDC}(X)) > \mathbb{Z}_2 \times \mathrm{Aut}(X)$.

Motivation

Stability of graphs

- S. Wilson, Unexpected symmetries of graphs, JCTB '08
- $\mathrm{Aut}(X)$ lifts to CDC(X) as $\mathbb{Z}_2 \times \mathrm{Aut}(X)$. Wilson determined conditions when $\mathrm{Aut}(\mathrm{CDC}(X)) > \mathbb{Z}_2 \times \mathrm{Aut}(X)$.
- One of these is: X is a $\mathbb{Z}_n$-crosscover of a smaller graph.

Motivation

Stability of graphs

- S. Wilson, Unexpected symmetries of graphs, JCTB '08
- $\mathrm{Aut}(X)$ lifts to CDC(X) as $\mathbb{Z}_2 \times \mathrm{Aut}(X)$. Wilson determined conditions when $\mathrm{Aut}(\mathrm{CDC}(X)) > \mathbb{Z}_2 \times \mathrm{Aut}(X)$.
- One of these is: X is a $\mathbb{Z}_n$-crosscover of a smaller graph.

(3,6)-fullerenes

Motivation

Stability of graphs

- S. Wilson, Unexpected symmetries of graphs, JCTB '08
- $\mathrm{Aut}(X)$ lifts to CDC(X) as $\mathbb{Z}_2 \times \mathrm{Aut}(X)$. Wilson determined conditions when $\mathrm{Aut}(\mathrm{CDC}(X)) > \mathbb{Z}_2 \times \mathrm{Aut}(X)$.
- One of these is: X is a $\mathbb{Z}_n$-crosscover of a smaller graph.

(3,6)-fullerenes

- Conjecture of Fowler, '95: spectrum $= [3, -1, -1, -1] \cup L \cup -L$

Motivation

Stability of graphs

- S. Wilson, Unexpected symmetries of graphs, JCTB '08
- $\mathrm{Aut}(X)$ lifts to CDC(X) as $\mathbb{Z}_2 \times \mathrm{Aut}(X)$. Wilson determined conditions when $\mathrm{Aut}(\mathrm{CDC}(X)) > \mathbb{Z}_2 \times \mathrm{Aut}(X)$.
- One of these is: X is a $\mathbb{Z}_n$-crosscover of a smaller graph.

(3,6)-fullerenes

- Conjecture of Fowler, '95: spectrum $= [3, -1, -1, -1] \cup L \cup -L$
- DeVos, Goddyn, Mohar, Šámal, JCTB '09 : Cayley sum graphs!

Motivation

Stability of graphs

- S. Wilson, Unexpected symmetries of graphs, JCTB '08
- $\mathrm{Aut}(X)$ lifts to $\mathrm{CDC}(X)$ as $\mathbb{Z}_2 \times \mathrm{Aut}(X)$. Wilson determined conditions when $\mathrm{Aut}(\mathrm{CDC}(X)) > \mathbb{Z}_2 \times \mathrm{Aut}(X)$.
- One of these is: X is a $\mathbb{Z}_n$-crosscover of a smaller graph.

(3,6)-fullerenes

- Conjecture of Fowler, '95: spectrum $= [3, -1, -1, -1] \cup L \cup -L$
- DeVos, Goddyn, Mohar, Šámal, JCTB '09 : Cayley sum graphs!
- Spectrum determined easily via complex irreducible characters (similarly as for Cayley graphs)

Motivation

Stability of graphs

- S. Wilson, Unexpected symmetries of graphs, JCTB '08
- $\mathrm{Aut}(X)$ lifts to $\mathrm{CDC}(X)$ as $\mathbb{Z}_2 \times \mathrm{Aut}(X)$. Wilson determined conditions when $\mathrm{Aut}(\mathrm{CDC}(X)) > \mathbb{Z}_2 \times \mathrm{Aut}(X)$.
- One of these is: X is a $\mathbb{Z}_n$-crosscover of a smaller graph.

(3,6)-fullerenes

- Conjecture of Fowler, '95: spectrum $= [3, -1, -1, -1] \cup L \cup -L$
- DeVos, Goddyn, Mohar, Šámal, JCTB '09 : Cayley sum graphs!
- Spectrum determined easily via complex irreducible characters (similarly as for Cayley graphs)

Develop the general theory of crosscovers

Stability of graphs

- S. Wilson, Unexpected symmetries of graphs, JCTB '08
- $\mathrm{Aut}(X)$ lifts to $\mathrm{CDC}(X)$ as $\mathbb{Z}_2 \times \mathrm{Aut}(X)$. Wilson determined conditions when $\mathrm{Aut}(\mathrm{CDC}(X)) > \mathbb{Z}_2 \times \mathrm{Aut}(X)$.
- One of these is: X is a $\mathbb{Z}_n$-crosscover of a smaller graph.

(3,6)-fullerenes

- Conjecture of Fowler, '95: spectrum $= [3, -1, -1, -1] \cup L \cup -L$
- DeVos, Goddyn, Mohar, Šámal, JCTB '09 : Cayley sum graphs!
- Spectrum determined easily via complex irreducible characters (similarly as for Cayley graphs)

Develop the general theory of crosscovers

- Interesting in its own right

Motivation

Stability of graphs

- S. Wilson, Unexpected symmetries of graphs, JCTB '08
- $\mathrm{Aut}(X)$ lifts to CDC(X) as $\mathbb{Z}_2 \times \mathrm{Aut}(X)$. Wilson determined conditions when $\mathrm{Aut}(\mathrm{CDC}(X)) > \mathbb{Z}_2 \times \mathrm{Aut}(X)$.
- One of these is: X is a $\mathbb{Z}_n$-crosscover of a smaller graph.

(3,6)-fullerenes

- Conjecture of Fowler, '95: spectrum $= [3, -1, -1, -1] \cup L \cup -L$
- DeVos, Goddyn, Mohar, Šámal, JCTB '09 : Cayley sum graphs!
- Spectrum determined easily via complex irreducible characters (similarly as for Cayley graphs)

Develop the general theory of crosscovers

- Interesting in its own right
- Certain irregular covers can be studied via abelian groups

Developing the general theory – Main questions

Developing the general theory – Main questions

Basics

Developing the general theory – Main questions

Basics

- Unique path-lifting via crossvoltages, action of $\pi(X, b)$

Developing the general theory – Main questions

Basics

- Unique path-lifting via crossvoltages, action of $\pi(X, b)$
- Number of components, connectedness

Developing the general theory – Main questions

Basics

- Unique path-lifting via crossvoltages, action of $\pi(X, b)$
- Number of components, connectedness
- Which crosscovers are regular/irregular as covers

Developing the general theory – Main questions

Basics

- Unique path-lifting via crossvoltages, action of $\pi(X, b)$
- Number of components, connectedness
- Which crosscovers are regular/irregular as covers
- Recognition of crosscovers

Developing the general theory – Main questions

Basics

- Unique path-lifting via crossvoltages, action of $\pi(X, b)$
- Number of components, connectedness
- Which crosscovers are regular/irregular as covers
- Recognition of crosscovers

Comparing symmetries of $\mathrm{Cr}(X)$ and X

Developing the general theory – Main questions

Basics

- Unique path-lifting via crossvoltages, action of $\pi(X, b)$
- Number of components, connectedness
- Which crosscovers are regular/irregular as covers
- Recognition of crosscovers

Comparing symmetries of $\mathrm{Cr}(X)$ and X

- Lifting and projecting automorphisms, Group extensions

Developing the general theory – Main questions

Basics

- Unique path-lifting via crossvoltages, action of $\pi(X, b)$
- Number of components, connectedness
- Which crosscovers are regular/irregular as covers
- Recognition of crosscovers

Comparing symmetries of $\mathrm{Cr}(X)$ and X

- Lifting and projecting automorphisms, Group extensions
- Decomposition of crosscovers

Developing the general theory – Main questions

Basics

- Unique path-lifting via crossvoltages, action of $\pi(X, b)$
- Number of components, connectedness
- Which crosscovers are regular/irregular as covers
- Recognition of crosscovers

Comparing symmetries of $\mathrm{Cr}(X)$ and X

- Lifting and projecting automorphisms, Group extensions
- Decomposition of crosscovers

And more

Developing the general theory – Main questions

Basics

- Unique path-lifting via crossvoltages, action of $\pi(X, b)$
- Number of components, connectedness
- Which crosscovers are regular/irregular as covers
- Recognition of crosscovers

Comparing symmetries of $\mathrm{Cr}(X)$ and X

- Lifting and projecting automorphisms, Group extensions
- Decomposition of crosscovers

And more

- Using tools from Linear algebra and Representaion theory

Developing the general theory – Main questions

Basics

- Unique path-lifting via crossvoltages, action of $\pi(X, b)$
- Number of components, connectedness
- Which crosscovers are regular/irregular as covers
- Recognition of crosscovers

Comparing symmetries of $\mathrm{Cr}(X)$ and X

- Lifting and projecting automorphisms, Group extensions
- Decomposition of crosscovers

And more

- Using tools from Linear algebra and Representaion theory
- Algorithmic and complexity aspects

Developing the general theory – Main questions

Basics

- Unique path-lifting via crossvoltages, action of $\pi(X, b)$
- Number of components, connectedness
- Which crosscovers are regular/irregular as covers
- Recognition of crosscovers

Comparing symmetries of $\mathrm{Cr}(X)$ and X

- Lifting and projecting automorphisms, Group extensions
- Decomposition of crosscovers

And more

- Using tools from Linear algebra and Representaion theory
- Algorithmic and complexity aspects
- Applications

$\mathrm{Cr}(X)$ connected: regular or irregular as a cover?

$\mathrm{Cr}(X)$ connected: regular or irregular as a cover?

X = bipartite

$\mathrm{Cr}(X)$ connected: regular or irregular as a cover?

X = bipartite

- A regular cover.

$\mathrm{Cr}(X)$ connected: regular or irregular as a cover?

X = bipartite

- A regular cover.
- $\mathrm{CT} = \Gamma$

$\mathrm{Cr}(X)$ connected: regular or irregular as a cover?

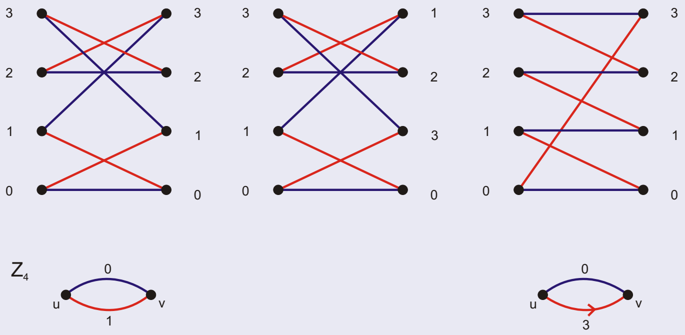
X = bipartite

- A regular cover.
- $\mathrm{CT} = \Gamma$
- Action of CT: $\tau_a\colon g \mapsto a + g$ and $\tau_{-a}\colon g \mapsto -a + g$

$\mathrm{Cr}(X)$ connected: regular or irregular as a cover?

X = bipartite

- A regular cover.
- $\mathrm{CT} = \Gamma$
- Action of CT: $\tau_a\colon g \mapsto a + g$ and $\tau_{-a}\colon g \mapsto -a + g$



$\mathrm{Cr}(X)$ connected: regular or irregular as a cover?

$\mathrm{Cr}(X)$ connected: regular or irregular as a cover?

X $\neq$ bipartite, $\exists$ odd closed walk $\zeta_W = 0$

$\mathrm{Cr}(X)$ connected: regular or irregular as a cover?

X $\neq$ bipartite, $\exists$ odd closed walk $\zeta_W = 0$

- Regular only if Γ elementary abelian.

$\mathrm{Cr}(X)$ connected: regular or irregular as a cover?

X $\neq$ bipartite, $\exists$ odd closed walk $\zeta_W = 0$

- Regular only if Γ elementary abelian.
- $\mathrm{CT} = \mathbb{Z}_2(\Gamma)$.

$\mathrm{Cr}(X)$ connected: regular or irregular as a cover?

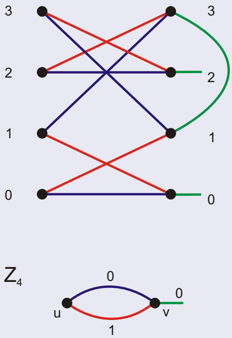
X $\neq$ bipartite, $\exists$ odd closed walk $\zeta_W = 0$

- Regular only if Γ elementary abelian.
- $\mathrm{CT} = \mathbb{Z}_2(\Gamma)$.
- Action of CT: $\tau_a \mapsto a + g$, where $2a = 0$

$\mathrm{Cr}(X)$ connected: regular or irregular as a cover?

X $\neq$ bipartite, $\exists$ odd closed walk $\zeta_W = 0$

- Regular only if Γ elementary abelian.
- $\mathrm{CT} = \mathbb{Z}_2(\Gamma)$.
- Action of CT: $\tau_a \mapsto a + g$, where $2a = 0$



$\mathrm{Cr}(X)$ connected: regular or irregular as a cover?

$\mathrm{Cr}(X)$ connected: regular or irregular as a cover?

X $\neq$ bipartite, no odd closed walk $\zeta_W = 0$

$\mathrm{Cr}(X)$ connected: regular or irregular as a cover?

X $\neq$ bipartite, no odd closed walk $\zeta_W = 0$

- A regular cover.

$\mathrm{Cr}(X)$ connected: regular or irregular as a cover?

X $\neq$ bipartite, no odd closed walk $\zeta_W = 0$

- A regular cover.
- $\mathrm{CT} \cong \{1, -1\} \ltimes \Gamma^0 \cong (\Gamma, *)$, where $g * h = \sigma_h \cdot g + h$

$\mathrm{Cr}(X)$ connected: regular or irregular as a cover?

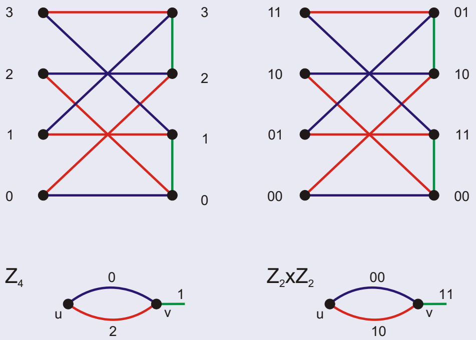
X $\neq$ bipartite, no odd closed walk $\zeta_W = 0$

- A regular cover.
- $\mathrm{CT} \cong \{1, -1\} \ltimes \Gamma^0 \cong (\Gamma, *)$, where $g * h = \sigma_h \cdot g + h$
- CT acts as $\tau_a \colon g \mapsto \sigma_g \cdot a + g$ on vertices reachable by trivial voltage walks of even length, and as τ_{-a} otherwise.

$\mathrm{Cr}(X)$ connected: regular or irregular as a cover?

X $\neq$ bipartite, no odd closed walk $\zeta_W = 0$

- A regular cover.
- $\mathrm{CT} \cong \{1, -1\} \ltimes \Gamma^0 \cong (\Gamma, *)$, where $g * h = \sigma_h \cdot g + h$
- CT acts as $\tau_a \colon g \mapsto \sigma_g \cdot a + g$ on vertices reachable by trivial voltage walks of even length, and as τ_{-a} otherwise.



Thank you!