

Rational quadrangles

Two cases should be run: $m = 84$ and $m = 240$. Also $m = 12 \cdot 23$ might be useful to test the general case

```
m = 12*23
K.<zeta> = CyclotomicField(m)
ii = zeta^(m/4)
ii^2
-1
```

```
sins = list ((zeta^k-zeta^(-k))/2/ii for k in range (0,m))
coss = list ((zeta^k+zeta^(-k))/2 for k in range (0,m))
```

Let us first verify the results of Myerson

```
dict = {}
for i in range (1,m/4):
    #if 12*i != m:
        for j in range (i,m/4):
            #if 12*j != m:
                key = sins[i]*sins[j]
                l = [[i,j]]
                if (dict.has_key(key)):
                    l = l + dict.get(key)
                dict.update ({ key: l })
```

```
for i in dict.values():
    if len(i) > 1:
        print list([[2*r[0]/m,2*r[1]/m] for r in i)
```

```
[[5/69, 1/6], [5/138, 32/69]]
[[8/69, 1/6], [4/69, 61/138]]
[[7/69, 1/6], [7/138, 31/69]]
[[17/69, 35/138], [1/6, 34/69]]
[[1/6, 5/23], [5/46, 9/23]]
[[1/6, 4/23], [2/23, 19/46]]
[[1/6, 16/69], [8/69, 53/138]]
[[1/6, 17/69], [17/138, 26/69]]
[[31/138, 19/69], [1/6, 31/69]]
[[14/69, 41/138], [1/6, 28/69]]
[[29/138, 20/69], [1/6, 29/69]]
[[16/69, 37/138], [1/6, 32/69]]
[[5/23, 13/46], [1/6, 10/23]]
[[1/6, 19/69], [19/138, 25/69]]
[[1/6, 20/69], [10/69, 49/138]]
[[1/6, 7/23], [7/46, 8/23]]
[[9/46, 7/23], [1/6, 9/23]]
[[4/23, 15/46], [1/6, 8/23]]
[[25/138, 22/69], [1/6, 25/69]]
[[1/6, 22/69], [11/69, 47/138]]
[[13/69, 43/138], [1/6, 26/69]]
[[11/69, 1/6], [11/138, 29/69]]
[[1/6, 14/69], [7/69, 55/138]]
[[3/23, 1/6], [3/46, 10/23]]
[[4/69, 1/6], [2/69, 65/138]]
[[1/23, 1/6], [1/46, 11/23]]
[[2/23, 1/6], [1/23, 21/46]]
[[1/69, 1/6], [1/138, 34/69]]
[[2/69, 1/6], [1/69, 67/138]]
[[1/6, 13/69], [13/138, 28/69]]
[[1/6, 6/23], [3/23, 17/46]]
[[11/46, 6/23], [1/6, 11/23]]
[[10/69, 1/6], [5/69, 59/138]]
```

We apply the same method, directly to the sin formula.

We do not consider the cases where $\alpha = \gamma/2$ and $\alpha = \gamma$

```
dict = {}
for alpha in range (1,m/2):
    for halfgamma in range (1,m/4):
        numer = (alpha-halfgamma+m)%m
        if (numer != 0 and numer != halfgamma):
            key = sins[numer]/sins[halfgamma]
            l = [[alpha,2*halfgamma]]
            if (dict.has_key(key)):
                l = l + dict.get(key)
            dict.update ({ key: l })
```

```
def print_abcd (alpha, beta, gamma, delta):
    if (2*alpha != gamma and 2*delta != beta and
        alpha+beta+gamma+delta > 2 and alpha + delta < 1 + beta and alpha + delta < 1
+ gamma and alpha + beta < 1 + delta and gamma+delta < 1 + alpha):
        cosa1 = (sins[m*delta/2] + coss[m*alpha/2]*sins[m*beta/2])/sins[m*alpha/2]/(1
- coss[m*beta/2])
        cosa2 = (sins[m*alpha/2] + coss[m*delta/2]*sins[m*gamma/2])/sins[m*delta/2]/(1
- coss[m*gamma/2])
        ncosa = N(cosa1).real()
        a = acos(ncosa)
        sinbsina = (coss[m*beta/2] - coss[m*gamma/2])/(coss[m*delta/2] -
coss[m*alpha/2])
        b = asin(sin(a)*N(sinbsina).real())
        print alpha, beta, gamma, delta, alpha+beta+gamma+delta-2, a, b, ("***" if
cosa1 in coss else "")
        # additional
        print "  -- ", 3*beta, 2*beta+gamma,beta+2*gamma,3*gamma,
2*alpha+beta,2*alpha+gamma,2*delta+beta,2*delta+gamma,alpha+delta+beta,alpha+delta+gamma
```

```
def print_solution (r,s):
    if (r[1] < s[1]):
        print_solution (s,r)
    if (r[1] > s[1]):
        alpha = 2*r[0]/m
        gamma = 2*r[1]/m
        delta = 2*s[0]/m
        beta = 2*s[1]/m
        print_abcd(alpha,beta,gamma, delta)
```

```
for v in dict.values():
    if (len(v) >= 2):
        for i in range (0,len(v)):
            for j in range (i+1,len(v)):
                print_solution (v[i], v[j])
```

```
17/23 35/69 68/69 29/69 15/23 1.17753571178719 1.32185624745424
-- 35/23 2 57/23 68/23 137/69 170/69 31/23 42/23 5/3 148/69
14/23 41/69 56/69 32/69 11/23 0.984668713748391 1.55463373973005
-- 41/23 2 51/23 56/23 125/69 140/69 35/23 40/23 5/3 130/69
25/46 34/69 52/69 17/46 11/69 0.632100145193459 0.953673509076181
-- 34/23 40/23 2 52/23 109/69 127/69 85/69 103/69 97/69 5/3
40/69 8/23 19/23 6/23 1/69 0.217134406084848 0.309286374353876
-- 24/23 35/23 2 57/23 104/69 137/69 20/23 31/23 82/69 5/3
16/23 9/23 18/23 25/69 16/69 0.835460029905981 0.973833223734608
-- 27/23 36/23 45/23 54/23 41/23 50/23 77/69 104/69 100/69
127/69
27/46 14/23 18/23 65/138 31/69 0.955769924307790 1.50944238224311
-- 42/23 2 50/23 54/23 41/23 45/23 107/69 119/69 5/3 127/69
33/46 12/23 22/23 59/138 43/69 1.14193340186121 1.36618689971593
-- 36/23 2 56/23 66/23 45/23 55/23 95/69 125/69 5/3 145/69
101/138 32/69 64/69 55/138 12/23 1.11994392955737 1.57044127710044
```

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142/69
73/138 38/69 50/69 19/46 5/23 0.712556197705323 1.09268822408556
-- 38/23 42/23 2 50/23 37/23 41/23 95/69 107/69 103/69 5/3
45/46 22/69 44/69 15/46 6/23 1.10927151239294 0.600903261357888
-- 22/23 88/69 110/69 44/23 157/69 179/69 67/69 89/69 112/69
134/69
17/23 11/23 22/23 28/69 40/69 1.15417917105749 1.46116905809968
-- 33/23 44/23 55/23 66/23 45/23 56/23 89/69 122/69 112/69
145/69
31/46 8/23 16/23 47/138 4/69 0.447764152685082 0.465619536124964
-- 24/23 32/23 40/23 48/23 39/23 47/23 71/69 95/69 94/69 118/69
37/69 12/23 17/23 9/23 13/69 0.674459890118708 1.02636360052127
-- 36/23 41/23 2 51/23 110/69 125/69 30/23 35/23 100/69 5/3
38/69 32/69 53/69 8/23 3/23 0.583810704074219 0.872463788998766
-- 32/23 39/23 2 53/23 36/23 43/23 80/69 101/69 94/69 5/3
25/46 44/69 50/69 67/138 9/23 0.898746637655459 1.41642207127520
-- 44/23 2 48/23 50/23 119/69 125/69 37/23 39/23 5/3 121/69
95/138 26/69 52/69 49/138 4/23 0.740252499865106 0.830997873038184
-- 26/23 104/69 130/69 52/23 121/69 49/23 25/23 101/69 98/69
124/69
31/46 38/69 62/69 61/138 13/23 1.07559047737723 1.45416980986092
-- 38/23 2 54/23 62/23 131/69 155/69 33/23 41/23 5/3 139/69
97/138 28/69 56/69 17/46 20/69 0.913160408002534 1.10486927761582
-- 28/23 112/69 140/69 56/23 125/69 51/23 79/69 107/69 34/23
130/69
29/46 40/69 58/69 21/46 35/69 1.01412704036939 1.54235718087376
-- 40/23 2 52/23 58/23 127/69 145/69 103/69 121/69 5/3 133/69
50/69 31/69 62/69 9/23 32/69 1.07986769790091 1.45993439505213
-- 31/23 124/69 155/69 62/23 131/69 54/23 85/69 116/69 36/23
139/69
47/69 25/69 50/69 8/23 8/69 0.618551435330068 0.668472668417209
-- 25/23 100/69 125/69 50/23 119/69 48/23 73/69 98/69 32/23
121/69
35/69 44/69 47/69 11/23 7/23 0.811229772055154 1.26613380201971
-- 44/23 45/23 2 47/23 38/23 39/23 110/69 113/69 112/69 5/3
49/69 29/69 58/69 26/69 8/23 0.978062275447058 1.22811787995828
-- 29/23 116/69 145/69 58/23 127/69 52/23 27/23 110/69 104/69
133/69
79/138 26/69 56/69 13/46 1/23 0.364307183951558 0.525993831612388
-- 26/23 36/23 2 56/23 35/23 45/23 65/69 95/69 85/69 5/3
13/23 28/69 55/69 7/23 5/69 0.457028286476838 0.668159577144076
-- 28/23 37/23 2 55/23 106/69 133/69 70/69 97/69 88/69 5/3
12/23 40/69 49/69 10/23 17/69 0.747531913202665 1.15410713425775
-- 40/23 43/23 2 49/23 112/69 121/69 100/69 109/69 106/69 5/3
43/46 20/69 40/69 43/138 8/69 0.769216793837388 0.400280397203477
-- 20/23 80/69 100/69 40/23 149/69 169/69 21/23 83/69 106/69
42/23
15/23 13/23 20/23 31/69 37/69 1.04436189245894 1.49815355072894
-- 39/23 2 53/23 60/23 43/23 50/23 101/69 122/69 5/3 136/69
71/138 14/23 16/23 21/46 19/69 0.780214530499817 1.21166345160004
-- 42/23 44/23 2 48/23 113/69 119/69 35/23 37/23 109/69 5/3
33/46 10/23 20/23 53/138 28/69 1.03300796037551 1.34595477738453
-- 30/23 40/23 50/23 60/23 43/23 53/23 83/69 113/69 106/69
136/69
35/46 1/3 35/69 19/46 1/69 0.257842546469737 0.133614932631850
-- 1 27/23 31/23 35/23 128/69 140/69 80/69 4/3 104/69 116/69
16/23 37/69 64/69 10/23 41/69 1.10803598681336 1.41023616636586
-- 37/23 2 55/23 64/23 133/69 160/69 97/69 124/69 5/3 142/69
20/23 1/3 40/69 26/69 11/69 0.848360181406776 0.448185321080710
-- 1 86/69 103/69 40/23 143/69 160/69 25/23 4/3 109/69 42/23
41/46 1/3 41/69 17/46 13/69 0.921867853274599 0.489813398150366
-- 1 29/23 35/23 41/23 146/69 164/69 74/69 4/3 110/69 128/69
39/46 1/3 13/23 53/138 3/23 0.768000339526329 0.403671386470583
-- 1 85/69 101/69 39/23 140/69 52/23 76/69 4/3 36/23 124/69
77/138 10/23 18/23 15/46 7/69 0.526977717835455 0.779313952668648
-- 30/23 38/23 2 54/23 107/69 131/69 25/23 33/23 91/69 5/3

22/23 7/23 14/23 22/69 13/69 0.961033592576064 0.510429872178184
-- 21/23 28/23 35/23 42/23 51/23 58/23 65/69 86/69 109/69 130/69
12/23 15/23 16/23 34/69 25/69 0.870129702733533 1.36808388050888
-- 45/23 2 47/23 48/23 39/23 40/23 113/69 116/69 5/3 118/69
103/138 34/69 68/69 19/46 44/69 1.18327297438966 1.35319850298191
-- 34/23 136/69 170/69 68/23 137/69 57/23 91/69 125/69 38/23
148/69
21/23 19/69 38/69 7/23 1/23 0.481066807938594 0.245068642300452
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122/69
45/46 1/3 15/23 47/138 7/23 1.17433578338064 0.642571671050466
-- 1 91/69 113/69 45/23 158/69 60/23 70/69 4/3 38/23 136/69
18/23 1/3 12/23 28/69 1/23 0.445534896189000 0.231487178920345
-- 1 82/69 95/69 36/23 131/69 48/23 79/69 4/3 35/23 118/69
43/46 1/3 43/69 49/138 17/69 1.05453731363896 0.567847235780651
-- 1 89/69 109/69 43/23 152/69 172/69 24/23 4/3 112/69 44/23
19/23 1/3 38/69 9/23 7/69 0.678131602768682 0.354846162636550
-- 1 28/23 33/23 38/23 137/69 152/69 77/69 4/3 107/69 122/69
37/46 1/3 37/69 55/138 5/69 0.574041607656377 0.299224068709525
-- 1 83/69 97/69 37/23 134/69 148/69 26/23 4/3 106/69 40/23
22/23 1/3 44/69 8/23 19/69 1.11569876139924 0.605409772347327
-- 1 30/23 37/23 44/23 155/69 176/69 71/69 4/3 113/69 134/69
21/23 1/3 14/23 25/69 5/23 0.990207947714251 0.529481653318870
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13/23 43/69 52/69 11/23 29/69 0.927207791760313 1.46345078466523
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